A statistical model for material removal prediction in polishing

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**ABSTRACT**

This paper develops analytically a statistical model for predicting the material removal in mechanical polishing of material surfaces (MS). The model was based on the statistical theory and the abrasive–MS contact mechanisms. The pad-MS and pad-abrasive-MS interactions in polishing were characterised by contact mechanics. Two types of active abrasive particles in the polishing system were considered, i.e., Type I – the particles that can slide and rotate between the pad and MS, and Type II – those embedded in the pad without a rigid body motion. Accordingly, the material removal is considered to be the sum of the contributions from the two types of abrasive interactions. It was found that the mechanical properties and microstructure of the polishing pad and polishing conditions have a significant effect on the material removal rate, such as the porosity and elastic modulus of the pad, polishing pressure, volume concentration of abrasives, particle size, pad asperity radius and pad roughness. It was also found that different types of active particles contribute quite differently to the material removal. When the mean particle radius is small, the material removal is mainly due to the Type II particles, but when the mean particle radius becomes large, the Type I particles remove more materials. The model predictions are well aligned with experimental results available in the literature and can be used for the material removal prediction in chemo-mechanical polishing if a proper treatment of the chemical effect is introduced.

**1. Introduction**

Mechanical polishing using abrasive slurry is a key finishing process in industry for producing a material surface (MS) of low surface roughness. Chemo-mechanical polishing (CMP) is a good example that is based on mechanical polishing, but introduces chemical reaction by adding chemicals to abrasive slurry to promote the material removal rate. The technique has been widely used in polishing glass, silicon and ceramic surfaces as well as in planarizing surfaces of inter-level dielectrics or inter-metal dielectrics during integrated circuit fabrication [1–5]. In a typical CMP process, a rotating material surface (MS) attached to a carrier is pressed against a rotating polishing pad in the presence of liquid slurry which contains abrasive particles with chemicals. The material removal of the process is generally due to the combination of erosion and abrasion. It is known that many variables such as applied normal force, relative velocity of the MS to the pad, pad properties (elastic modulus, hardness, etc.) and slurry characteristics, have profound influences on the material removal mechanically. The fundamental mechanisms of the material removal in the process are very complicated and have not been well understood, because of the statistical nature of the surfaces in contact such as the random distributions of the surface asperities and abrasive particles.

In the literature, the modelling of the material removal in CMP processes can be generally classified into two categories. One was based on fluid hydrodynamics. For example, Runels et al. [6,7] obtained the wear rate by numerically solving the Navier–Stockes equation. Sundrarajian et al. [8] studied the removal rate based on the lubrication and mass transport models, in which slurry erosion was considered a main mechanism. The other group of the modelling methods was based on the theory of contact mechanics. Since this approach is more plausible to describe experimental observations, it has been widely accepted and investigated. Larsen-basse and Liang [9] concluded that material removal in CMP is due to particle abrasion. There are also some similar investigations. Luo and Dornfeld [10] investigated the abrasion mechanism in solid–solid mode of the CMP process based on a number of assumptions: plastic wafer–abrasive and pad–abrasive contacts, normal distribution of abrasive size and periodic roughness of pad surface. They extended the model as a function of the abrasive weight concentration [11] and then further used it to explain the effects of abrasive size distribution [12]. However, these models were based on the assumption of periodic roughness of pad surface. From the perspective of pad modelling, abrasive behaviour and distribution effects of abrasive, Wang et al. [13] presented three models for material removal to try to understand how particle properties in conjunction with pad information influence material removal rate.

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### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>nominal area of contact between pad and material surface</td>
</tr>
<tr>
<td>$A_1$</td>
<td>total contact area by Type I particles</td>
</tr>
<tr>
<td>$A_2$</td>
<td>total contact area by Type II particles</td>
</tr>
<tr>
<td>$A_c$</td>
<td>total contact area in a polishing process</td>
</tr>
<tr>
<td>$A_d$</td>
<td>total direct contact area between the pad and material surface</td>
</tr>
<tr>
<td>$\tilde{A}_1$</td>
<td>area of asperity contact due to Type I particles</td>
</tr>
<tr>
<td>$\tilde{A}_2$</td>
<td>area of asperity contact due to Type II particles</td>
</tr>
<tr>
<td>$d$</td>
<td>separation of the reference planes of Surface 1 and Surface 2 (pad)</td>
</tr>
<tr>
<td>$E_2$</td>
<td>elastic modulus of a pad</td>
</tr>
<tr>
<td>$E'$</td>
<td>composite elastic modulus $E' = E_2/(1 - v_2^2)$</td>
</tr>
<tr>
<td>$G_1$</td>
<td>wear volume of the material due to Type I particles</td>
</tr>
<tr>
<td>$G_2$</td>
<td>wear volume of the material due to Type II particles</td>
</tr>
<tr>
<td>$G$</td>
<td>wear volume of the material by an individual active particle</td>
</tr>
<tr>
<td>$H_w$</td>
<td>hardness of a workpiece material</td>
</tr>
<tr>
<td>$K_r$</td>
<td>wear coefficient</td>
</tr>
<tr>
<td>$M_1$</td>
<td>material removal rate due to Type I particles</td>
</tr>
<tr>
<td>$M_2$</td>
<td>material removal rate due to Type II particles</td>
</tr>
<tr>
<td>$M_r$</td>
<td>total material removal rate</td>
</tr>
<tr>
<td>$p_0$</td>
<td>polishing pressure</td>
</tr>
<tr>
<td>$R_p$</td>
<td>average asperity radius of a pad</td>
</tr>
<tr>
<td>$P_1$</td>
<td>total contact force on Type I particles</td>
</tr>
<tr>
<td>$P_2$</td>
<td>total contact force on Type II particles</td>
</tr>
<tr>
<td>$P_d$</td>
<td>direct contact force between the pad and material surface</td>
</tr>
<tr>
<td>$P_1$</td>
<td>contact force of Type I particles</td>
</tr>
<tr>
<td>$P_2$</td>
<td>contact force of Type II particles</td>
</tr>
<tr>
<td>$r$</td>
<td>particle radius</td>
</tr>
<tr>
<td>$t$</td>
<td>polishing time</td>
</tr>
<tr>
<td>$u_r$</td>
<td>mean particle radius</td>
</tr>
<tr>
<td>$V$</td>
<td>pad/material sliding velocity</td>
</tr>
<tr>
<td>$x$</td>
<td>particle volume concentration of slurry</td>
</tr>
<tr>
<td>$z_2$</td>
<td>asperity height of Surface 2 (pad)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>porous coefficient</td>
</tr>
<tr>
<td>$\eta_v$</td>
<td>number of particles per unit volume of slurry</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>surface density of asperity on Surface 2 (pad)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>standard deviation of particle radius</td>
</tr>
<tr>
<td>$\phi_\alpha$</td>
<td>standard deviation of pad asperity height</td>
</tr>
<tr>
<td>$\phi_r(t)$</td>
<td>probability density function of particle size</td>
</tr>
<tr>
<td>$\phi(z_2)$</td>
<td>probability density function of pad asperity height</td>
</tr>
<tr>
<td>$v_2$</td>
<td>Poisson's ratio of a pad</td>
</tr>
</tbody>
</table>

Zhao and Chang [14] studied the material removal rate based on the elastic–plastic micro-contact mechanics and abrasion wear, where the chemical effect was claimed to have been formulated by introducing a density ratio of a chemical thin film. Oh and Seok [15] proposed a model for silicon dioxide CMP based on a multi-scale mechanical abrasion consideration and coupled with the effect of the slurry chemical diffusion. Bozkaya and Mutfu [16] investigated the material removal with two-body pad–wafer and three-body pad–abrasive–wafer contacts, and introduced a thin passivated layer on the wafer surface to take into account the effect of chemical reactions between slurry and wafer. Some researchers have also studied the wear mechanisms and material removal rates in CMP processes based on the combination of the above two approaches, i.e., contact mechanics and fluid hydrodynamics [17–20]. To our knowledge, most existing studies only consider the particles embedded in the pad as the active particles. In reality, however, many active particles also slide and rotate between the pad and wafer, as pointed out by Zhang and Tanaka [21]. Therefore, some critical questions naturally arise: How do the two types of active particles contribute to the material removal and how do their contributions vary with the change of polishing conditions when the particle size and distribution are random?

This paper will try to answer these questions by developing a statistical model for predicting the material removal in mechanical polishing where both the polishing pad surface and the particle size are random. The investigation will be based on the abrasion wear and contact mechanics of pad–MS and pad–abrasive–MS interactions.

### 2. Modelling

Assume that the material removal from a surface is caused by abrasion wear by the abrasive particles in polishing slurry. The polishing pressure applied on the MS, $p_0$, is carried by two kinds of contacts. One is the three-body contact of the pad, the abrasive particles and the MS (particle contact); and the second is the two-body contact directly between the pad and the MS (direct contact). In the scenario of the particle contact, the particles can be divided into two types, i.e., Type I – the particles can slide and rotate between the pad and MS, and Type II – the particles are embedded in the polishing pad so that they could not have any rigid body motion. The aim of our modelling below is to formulate the material removal rate (MRR) considering the key parameters including the porosity and elastic modulus of polishing pad, pressure, abrasive volume concentration and size, and pad asperity radius and roughness. In addition to the above, the modelling below will consider the random nature of the particle size and polishing pad surface. Thus the model to be developed will be statistical, which will better mimic a real polishing process. The model will also avoid the problems in the literature which mostly assume a uniform particle size and are based on a deterministic material removal mechanism in the modelling. However, as has been validated by the previously in the literature [10–14], the formulation in the present work will consider that the contact is static and the polishing pressure is constant.

#### 2.1. Particle contact and direct contact

**2.1.1. Modelling the particle contact**

As discussed previously, there are two types of particle contacts. The Type I particles can slide and rotate between the pad and MS. For convenience, these particles can be regarded as the additional asperities to define a new MS. If we assume that the particle radius (hence the asperity radius) is $r$, then the contact in this case becomes that between the new MS surface and the pad surface—the contact between two rough surfaces, as illustrated in Fig. 1. It should be noted that the asperity height of the pad is less than the separation $d$ of the reference planes of the two rough surfaces. The contact...
between the two rough surfaces can be generally described by the Greenwood-Tripp model [22], as briefed below. Assume that the new rough surface is Surface 1 and that of the pad is Surface 2. If the asperity shapes of the two rough surfaces are \( y_1(s) = s^2 / 2r \) and \( y_2(s) = s^2 / 2R_p \), where subscript \( i \) \((i = 1, 2)\) denotes Surface \( i \), \( R_p \) is the average asperity radius of the pad, and \( s \) is the local coordinate of the asperity. The pairs of asperities are not aligned generally. The contact point of the asperity pair on the two surfaces is situated at a distance \( r_1 = \rho(r + R_p) \) from the centre of the asperity on Surface 1 and a distance \( r_2 = \rho R_p(r + R_p) \) from the centre of the asperity on Surface 2, in which \( \rho \) is the distance of the asperities of the two surfaces. Based on the Hertzian theory, the asperity contact behaviour can be expressed as follows:

\[
\tilde{\lambda}_1 = \pi R \varepsilon
\]

\[
\tilde{P}_1 = \frac{4}{3} E^R R^{3/2} \rho^{1/2}
\]

where the equivalent radius \( R \) and the composite elastic modulus \( E \) are given by \( 1/R = 1/r + 1/R_p \) and \( 1/E = (1 - \nu_i^2)/E_i + (1 - \nu_r^2)/E_r \), respectively. The interference at the contact is \( w = 2r_2 - d - r_1^2 / 2R_p = 2r + 2d - d - r_2^2 / 2(r + R_p) \), in which \( 2r \) is the asperity height of Surface 2. Since a polishing pad is usually much softer than the abrasive particle and workpiece materials, the composite elastic modulus can be simplified to \( E = E_2 / (1 - \nu_2^2) \), where \( E_2 \) and \( \nu_2 \) are the solid pad elastic modulus and Poisson’s ratio, respectively. Assume that the abrasive asperities are spherical and the probability density function of the particle size is \( \phi_r(r) \). Then, the average volume of the abrasive particle is \( \int_0^\infty \frac{x}{4} / 3 \pi r^2 \phi_r(r) dr \). On the basis of the definition of the abrasive volume concentration, the number of particles per unit volume of the slurry, \( n_\nu \), can be computed by the following expression,

\[
\eta_\nu = \int_0^\infty \frac{x}{4} / 3 \pi r^2 \phi_r(r) dr
\]

where \( x \) is the abrasive volume concentration of the slurry. With \( \eta_\nu \) defined, the number of asperities in the range \( r \) to \( r + dr \) situated between \( \rho \) and \( \rho + dr \) from one asperity on Surface 2 can be calculated as \( 2n_\nu \rho d \phi_r(r) dr \). There are \( n_\nu A_0 \phi_r(z_2) dz_2 \) asperities on Surface 2 with heights between \( z_2 \) and \( z_2 + dz_2 \), where \( n_\nu \) is the surface density of asperity on Surface 2, \( \phi_r(z_2) \) is the probability density function of the pad asperity height and \( A_0 \) is the nominal area of contact between pad and MS. Therefore, the expected number of contacts can be obtained as

\[
n_{c1} = 2n_\nu \eta_\nu A_0 \int_0^\infty \int_{z_2}^{z_2 + dz_2} \int_{r_1}^{r_2} \rho \phi_r(z_2) \phi_r(r) dr dz_2 dr\]

Similarly, based on the statistical theory, the expected total area \( A_1 \) and force \( P_1 \) associated with the Type I contact are

\[
A_1 = 2n_\nu \eta_\nu A_0 \int_0^\infty \int_{z_2}^{z_2 + dz_2} \int_{r_1}^{r_2} \tilde{A}_1 \rho \phi_r(z_2) \phi_r(r) dr dz_2 dr\]

\[
P_1 = 2n_\nu \eta_\nu A_0 \int_0^\infty \int_{z_2}^{z_2 + dz_2} \int_{r_1}^{r_2} \tilde{P}_1 \rho \phi_r(z_2) \phi_r(r) dr dz_2 dr
\]

It should be pointed out that here \( \rho r_1 (r + R_p) \leq 2r \) and \( z_2 \leq d \). Then, substituting Eq. (1) into Eq. (4) results in

\[
A_1 = 2n_\nu \eta_\nu A_0 \int_0^\infty \int_{z_2}^{z_2 + dz_2} \rho \phi_r(z_2) \phi_r(r) r d r dz_2 dr
\]

\[
\tilde{H}_1 = 32 / 3 \pi n_\nu \eta_\nu A_0 \int_0^\infty \int_{z_2}^{z_2 + dz_2} \rho \phi_r(z_2) \phi_r(r) r d r dz_2 dr
\]

Now let us consider the Type II particles which are embedded in the pad with a rigid body motion. Again the asperity height of the pad is larger than \( d \). Assume that the particle-pad deformation is elastic. As the indentation into the MS by an active particle is very small [23] and is negligible compared with that into the pad, for simplicity we assume that the indentation into the pad is \( 2r \). This gives rise to

\[
\tilde{A}_2 = 2\pi r
\]

\[
\tilde{P}_2 = \frac{4}{3} E^R R^{3/2} (2r)^{3/2}
\]

The area that the particles could be embedded in the pad is \( \eta_\nu A_0 \int_0^\infty \rho \phi_r(z_2) dz_2 \). Thus the expected number of the contacts in this case can be described by

\[
n_{c2} = \eta_\nu \rho A_0 \int_0^\infty \rho \phi_r(z_2) dz_2 \int_0^\infty 2\rho \phi_r(r) dr
\]

The expected total area \( A_2 \) and force \( P_2 \) for the Type II contact are therefore

\[
A_2 = 4\pi n_\nu \eta_\nu A_0 \int_0^\infty (z_2 - d) \phi_r(z_2) dz_2 \int_0^\infty r^2 \phi_r(r) \phi_r(r) dr
\]

\[
P_2 = \frac{4}{3} \pi n_\nu A_0 \int_0^\infty (z_2 - d)^{3/2} \phi_r(z_2) dz_2 \int_0^\infty (2r)^{3/2} \sqrt{r^2 \phi_r(r) \phi_r(r)} dr
\]

2.1.2. Modelling the direct contact

Unlike the scenario of particle contact discussed in the last section, in the characterisation of the direct contact, the MS can be regarded as a smooth plane because the MS is much smoother than the polishing pad. Thus the GW model can be applied [24]. If there is no particle, the total contact area and force between the pad and MS are

\[
A_d = \eta_\nu A_0 \int_0^\infty (z_2 - d) \phi_r(z_2) dz_2
\]

\[
P_d = \frac{4}{3} \pi n_\nu A_0 \int_0^\infty (z_2 - d)^{3/2} \phi_r(z_2) dz_2
\]

where \( \eta_r = a \varepsilon \) and \( \alpha (0 < \alpha < 1) \) is the porosity coefficient introduced to capture the effect of the porous surface structure of the pad (the smaller the \( \alpha \), the more porous of the pad). When there are embedded particles on the pad surface, i.e., Type II particles, it is reasonable to assume that the actual pressure remains the same as that in the case without a particle. Therefore, the real direct contact area \( A_d \) and force \( P_d \) in this case are

\[
A_d = \eta_\nu A_0 \int_0^\infty (z_2 - d) \phi_r(z_2) dz_2 - A_2
\]

\[
P_d = \frac{4}{3} \pi n_\nu A_0 \int_0^\infty (z_2 - d)^{3/2} \phi_r(z_2) dz_2 - A_d
\]

It should be pointed out that physically \( A_d \geq 0 \).

The total contact area \( A_c \) between MS and pad with abrasive particles can be obtained by \( A_c = A_d + (P_1 + P_2) / H_w \), where \( H_w \) is the hardness of the workpiece material. Generally, \( A_c \) is dominated by \( A_2 \) and the force balance equation can be obtained as follows,

\[
P_1 + P_2 + P_d = \rho \phi_A A_0
\]

From Eq. (11), the separation \( d \) can be calculated.

2.2. Material removal

As aforementioned, the material removal from the materials is due to abrasion wear by both the Type I and Type II particles. Consider a single particle contact between the pad and MS illustrated in Fig. 2. Based on the wear mechanism, the wear volume of the material by an individual active particle is

\[
C_K = \frac{1}{2} \pi d^2 \varepsilon
\]
where $S$ is the cross-sectional area of the worn groove in the MS generated by the active particle, i.e., the shaded area in Fig. 2. $V$ is the pad/material sliding velocity, $t$ is the polishing time and $K$ is the wear constant. $S$ can be determined by

$$S = r_a \delta_w$$  \hspace{1cm} (13)$$

where $r_a$ is the radius of the contact area between the active particle and the MS and $\delta_w$ is the indentation depth of the particle into the MS. It is noticed that $r_a^2 = 2r_0$, and $P_p = H_w \pi r_a^2$, where $P_p$ is the particle contact force and can be either $P_1$ or $P_2$. Then Eq. (13) can be rewritten as

$$S = \left( \frac{P_p}{\pi H_w} \right)^{3/2} \frac{1}{2T}$$  \hspace{1cm} (14)$$

By using Eqs. (12) and (14), the wear of the material by Type I contact, $G_1$, is

$$G_1 = \frac{K \Delta \alpha}{H_w^{3/2}} K_1$$  \hspace{1cm} (15)$$

where

$$M_1 = \frac{8 \pi \eta_p \eta_r}{(H_w)^{7/2}} \left( \frac{4E^2}{3\pi} \right)^{3/2} \int_0^d \int_{d-x}^{d-x} \left( r \frac{r_p}{r + r_p} \right)^{3/4} (r + r_p)(z_2 - z_1)^{13/4} \phi(z_2) \phi(r) \, dz_2 \, dr$$  \hspace{1cm} (16)$$

For the Type II contact, the wear of the material $G_2$ is similar to Eq. (15) except that $M_1$ in the equation is replaced by $M_2$ below

$$M_2 = \frac{8 \pi \eta_p \eta_r}{(H_w)^{7/2}} \left( \frac{4E^2}{3\pi} \right)^{3/2} \int_0^d (z_2 - z_1) \phi(z_2) \, dz_2 \int_0^\infty \left( r \frac{r_p}{r + r_p} \right)^{3/4} (2r)^{3/4} \phi(r) \, dr$$  \hspace{1cm} (17)$$

Then the total MRR $M_t$ can be derived from the total wear of the material in the process, i.e., the summation of the wear by both Type I and Type II contacts, given by

$$M_t = G_1 + G_2 = \frac{K \Delta \alpha}{H_w^{3/2}}$$  \hspace{1cm} (18)$$

where $M = M_1 + M_2$. It can be seen from Eq. (18) that with given $K$, $V$ and $H_w$, the MRR is solely determined by $M$.

It should be noted that the determination of the effective hardness is out of the scope of the present study.

### 3. Results and discussion

As a special case, assume that both $\phi(z_2)$ and $\phi_0(r)$ are Gaussian, i.e.,

$$\phi(z_2) = \exp(-z_2^2/2\sigma_z^2)$$

and $\phi_0(r) = \exp(-r^2/2\sigma_r^2)$, where $\sigma_z$ is the standard deviation of pad asperity height, $u_1$ is the mean particle radius and $\sigma_r$ is the standard deviation of particle radius. The MRR can therefore be obtained by the model of Eq. (18). With this model, in addition to the effects of the porosity and elastic modulus of the pad, polishing pressure, volume concentration of abrasives, mean particle radius, pad asperity radius and pad roughness, we can also investigate the contributions of these parameters to $M_1$ and $M_2$ individually. The primary parameter values used for the model calculation and their variation ranges used for examining the effect of each parameter are listed in Table 1. Hence, if it is not further specified, the parameter values in the model calculation will be from this table. Since $K$, $V$ and $H_w$ are constants in a given polishing system, we will discuss $M$ in Eq. (18) instead of $M_t$. For convenience, we will still refer to $M$ as MRR in the discussion.

#### 3.1. Effect of polishing pressure

The effect of pressure $p_0$ on $M$ is shown in Fig. 3 when the property of the polishing pad, elastic modulus $E_2$ and porosity coefficient $\alpha$ change. It can be seen that the MRR is nearly proportional to the

![Fig. 3. Variation of material removal rate (M) with polishing pressure and pad properties.](image-url)
applied pressure. Both the pad elastic modulus and porosity alter the MRR. A larger $E_2$ (hence a harder pad) or a smaller $\alpha$ (hence a more porous pad) will lead to a larger MRR. This is in agreement with the observations by Bozkaya and Muftu [16], and can be understood as follows: For a harder pad, the particle contact force becomes larger and then the indentation into the MS by an active particle increases, causing the MRR to increase. While with a more porous pad, the deformation of the pad asperities in polishing can be larger, i.e., the separation $d$ becomes smaller, leading to more active particles in contact with the workpiece and in turn bringing about a greater MRR.

Fig. 4 shows the effect of the polishing pressure on $M_1$ and $M_2$, indicating the contributions of Type I and Type II to the total MRR. It can be found that the MRR is dominated by Type II particles. $M_2$ increases with the polishing pressure while $M_1$ increases first but then decreases with the polishing pressure. It is easy to understand that a larger polishing pressure will lead to greater deformation in the polishing system. As a result, the number of the active particles becomes large. When the pressure increases to a certain extent, the Type I particles can become Type II. It is therefore reasonable that the variation of $M_1$ with $p_0$ is peaked at certain value of $p_0$.

The total contact area between MS and pad with abrasive particles predicted by the model is 1.89% at $p_0 = 0.01$ MPa and 4.54% at $p_0 = 0.025$ MPa. They are smaller than the corresponding 5% and 7.19% experimentally measured [25]. This is reasonable because Zhang, Biddut and Ali [25] used a colour coating method to record the contact area, which has a low resolution, cannot accurately eliminate the small valleys, and hence overestimates the total contact area. Using their experimental data at polishing pressures of $p_0 = 0.01$ MPa, $p_0 = 0.02$ MPa and $V = 0.17$ m/s [26], the average wear coefficient in Eq. (18) is determined to be $K = 3.545$. The model then predicts that the MRR is 0.0575 $\mu$m/min under the polishing condition of $H_p = 12.7$ GPa, $V = 0.143$ m/s and $p_0 = 0.01$ MPa, which is in very good agreement with the experimental result of 0.075 $\mu$m/min [25] under the same polishing condition. Fig. 5 shows the comparison of the model-predicted MRR with another experiment [27], where the normalized MRR is defined as the MRR normalized by the largest MRR in the range of examination/consideration, i.e., $0 \leq p_0 \leq 34,300$ Pa. In the calculation using the model, the volume concentration of abrasives $x$ is the same as that used in the experiment and all the parameters are as follows: $x = 2.1\% / 2.5$, $\eta = 25$ nm, $\sigma_r = 6.25$ nm, $\eta_p = 2 \times 10^{-4}$ $\mu$m, $R_p = 50$ $\mu$m, $\sigma_p = 5$ $\mu$m, $E_2 = 10$ MPa, $\eta_v = 0.49$ and $\alpha = 0.25$. In the experiment, MRR is about 3.1 $\mu$m/h when $x = 0$ (i.e., when the MRR was solely due to the etching by slurty chemicals). However, such a chemical effect is not directly included in the model. Hence, to make a reasonable comparison between the model prediction and the experimental result, it is necessary to subtract the MRR due to etching at $x = 0$ (i.e., 3.1 $\mu$m/h) from all the MRR values experimentally measured before normalisation. It can be seen from Fig. 5 that the model gives excellent predictions. Furthermore, we can also observe that MRR is almost proportional to the applied pressure.

### 3.2. Effect of abrasive volume concentration

The effect of the abrasive volume concentration $x$ on $M$ is shown in Fig. 6, when the pad elastic modulus $E_2$ and porous coefficient $\alpha$ change. It can be seen that $M$ increases with the increasing abrasive volume concentration, but eventually approaches a steady state. This is because an increase in abrasive concentration will first result in an increase in the number of active particles. However, a saturation state will be reached when the increase of $x$ makes the direct contact area of pad-workpiece diminish, as shown in Fig. 7. From Eq. (10), such saturation $x$ is $\int_0^\infty r^3\phi_1(r)/3dr/\int_0^\infty r^3\phi_1(r)/2(1 + R_p/r_0)\phi_1(r)dr$, which depends on the particle size, and pad topography.

The effect of the abrasive volume concentration $x$ on $M_1$ and $M_2$ is shown in Fig. 8. Similarly, Type II contact dominates the MRR. Both $M_1$ and $M_2$ increase with $x$ due to the increasing number of the active particles.
Fig. 7. The direct contact area ratio vs. x.

Fig. 8. The effect of the abrasive volume concentration on $M_1$ and $M_2$.

Fig. 9. Variation in the normalized MRR with the abrasive volume concentration.

It can be seen that the model predictions are in excellent agreement with the experimental results.

3.3. Effects of mean particle radius and pad porosity

In addition to the effect of abrasive volume concentration, abrasive particle size in polishing slurry is also of vital importance to the material removal. The effect of the mean particle radius $u_0$ on $M$, as shown in Fig. 10(a), is more complicated. $M$ reaches a peak at a critical $u_0$, and then decrease. With a larger $u_0$, the mean volume of the abrasive particle is larger so that the number of particles per unit volume in the slurry is smaller. As a result, the number of the active particles in contact with the workpiece becomes smaller. On the other hand, the mean particle contact force increases with the increasing mean particle radius. In coupling these effects, hence, the MRR can decrease or increase with the mean particle radius, depending on the relative influence of the individual. When the mean particle radius $u_0$ is small (<100 nm), MRR drops with the increase of $u_0$, as shown in Fig. 10(b). This means that in this particle size range, the effect of the particle size on the number of the active particles is stronger than that on the mean particle contact force. We noticed that some earlier reports [28,29] on the effect of particle size show contradictory conclusions: Xie et al. [28] found that MRR increases with the particle size, while Biehlmann et al. [29] concluded the opposite. The results from the present model clarify this issue: It is the particle size range that makes the MRR variation different, but this range is related to other polishing parameters.
Moreover, the analysis using our model emphasises that the relative influence of individual factors determines the coupled effect on MRR. The pad porosity plays an important role on the MRR. When the mean particle radius \( u_t \) is smaller than the critical mean particle, the smaller the porous coefficient \( \alpha \), the flatter the variation of MRR. This can be explained as follows. When the mean particle radius \( u_t \) increases, the number of active particles in Type II drops rapidly. Thus \( M_2 \) decreases with \( u_t \), and finally reduces to zero, as shown in Fig. 11. The effect of the porous coefficient is mainly on the MRR achieved by Type II particles. The more porous the pad is, the more obvious the effect becomes. However, this effect becomes weak when \( u_t \) increases. As a result, \( M_2 \) decreases more significantly when the pad is more porous. On the other hand, with the increase of \( u_t \), the number of Type I particles first increases and then decreases as has been discussed previously. Therefore, \( M_1 \) first increases with \( u_t \) and then decreases with it. In a certain range, \( M_1 \) increases more quickly due to the additional effect of the particle size on the particle contact force. Thus, in this situation, the variation of the total MRR is slower for more porous pad. It can be found from Fig. 11 that Type II particles dominate the MRR when \( u_t \) is smaller but Type I particles play a more important role when \( u_t \) is large.

### 3.4. Effect of pad asperity radius

The surface topography of a rough pad has a strong influence on the contact behaviour in polishing, such as the contact area and contact force, as demonstrated in Fig. 12. Therefore, it is very much worthwhile to understand the effect of the surface topography on \( M \). The results with respect to the pad asperity radius \( R_p \) are shown in Fig. 13 with different elastic modulus \( \varepsilon_2 \) and porous coefficient \( \alpha \) of the pads. It can be seen that the MRR can be enhanced by using larger pad asperity radius. Similar trend is found in [16]. Fig. 14 shows the effect of \( R_p \) on \( M_1 \) and \( M_2 \). Clearly, Type II particles dominate the MRR. Both \( M_1 \) and \( M_2 \) increase with the pad asperity radius, and this can be explained by the variation in the number of active particles. As \( R_p \) increases, the area that the Type II particles could be embedded in the pad increases, leading to the number of the Type II particles to increase.
3.5. Effect of pad roughness

Another quantity related to pad topography, pad roughness, is also influential to the MRR of a polishing system. Fig. 15 shows the effect of the standard deviation of pad asperity height $\sigma_p$, which is a description of pad roughness. A smaller $\sigma_p$ means a smoother pad, capable of holding a greater number of active particles. Therefore, it is reasonable to see from Fig. 15 that $M$ decreases with increasing $\sigma_p$. It is also found that the larger the $E_2$ or the smaller the $\alpha$ is, the larger the MRR becomes. Using different pad conditioning methods, Park and Jeong [30] investigated experimentally the effect of pad topographies with $\sigma_p = 4.94 \, \mu m$ and $\sigma_p = 2.96 \, \mu m$ using silica slurry. They reported that the average MRR achieved using the $\sigma_p = 4.94 \, \mu m$ pad was 10% lower than that by using the $\sigma_p = 2.96 \, \mu m$ pad. Our model predicts that the MRR difference using these two pads is 10.5%, which is in excellent agreement with the experiment [30]. In the model prediction, we have used the same polishing conditions described by [30] quantified by Bozkaya and Muftu [16], except that we used the correct density value of silica in calculation. Fig. 16 shows the effect of the pad roughness on $M_1$ and $M_2$, individually demonstrating that the Type II particles dominate the MRR but that both $M_1$ and $M_2$ decrease with increasing the pad roughness.

![Fig. 14. Effect of pad asperity radius on $M_1$ and $M_2$.](image1.png)

![Fig. 16. Effect of the pad roughness on $M_1$ and $M_2$.](image2.png)

4. Conclusions

This paper presents a statistical model for the prediction of material removal rate in mechanical polishing of materials. The contact interactions in the polishing process are characterised by the contact mechanics of pad–MS and pad–abrasive–MS interactions. In the pad–abrasive–MS contact, two types of active abrasive particles are considered, i.e., Type I – the particles able to slide and rotate between the pad and MS, and Type II – those embedded in the pad without a rigid body motion. The model has been comprehensively verified by experimental results in the literature. This predictive model enables a detailed investigation into the effects of many key polishing parameters on the MRR, including polishing pressure, abrasive volume concentration and abrasive particle size of polishing slurry, and elastic modulus, surface topography and porosity of a polishing pad.

The analysis with the aid of this model has led to the following major findings:

1. MRR increases monotonically with the abrasive volume concentration. However, there is a critical volume concentration beyond which a saturation state will be reached. In other words, when the abrasive volume concentration reaches a critical value, MRR will become a constant. Pad surface topography and particle size distribution will influence the critical volume concentration value in a polishing process.

2. MRR varies with the mean particle radius. It is the range of the particle size distribution that makes the MRR vary in a different manner, increasing or decreasing. This elucidates the conflicting experimental conclusions in the literature. Our analysis also demonstrates that the relative influence of individual polishing parameters determines the coupled effect on MRR.

3. When the mean particle radius is small, MRR is mainly due to Type II particles; however, when the mean particle radius becomes large, MRR is mainly due to Type I particles.

4. MRR increases with increasing the pad asperity radius and decreases with the increase of the pad roughness. A harder or a more porous pad will lead to a larger MRR.

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References


